Extra Homework 01 Building Definite Integrals

Density of one-dimensional objects, like thin wires, is measured in units of mass per unit length. For example, the density of a simple wire might be $10\frac{\text{g}}{\text{cm}}$. Such a wire is said to have constant density. Other wires can have densities that change as you move along the wire. For example, a wire that is made of one metal at one end and progressively changes to a different metal at the other end might have a density function of $\delta(x) = 10x\,\frac{\text{g}}{\text{cm}}$ with domain $1 \leq x \leq 3$. Such a wire would have density of $\delta(1) = 10\frac{\text{g}}{\text{cm}}$ at one end, $\delta(3) = 30\frac{\text{g}}{\text{cm}}$ at the other, and $\delta(2.5) = 25\frac{\text{g}}{\text{cm}}$ three quarters of the way along.

1. Use the fact that, for a one-dimensional object with **constant** density $k\,\frac{\text{g}}{\text{cm}}$ and length L cm, the total mass is kL grams, to find a definite integral formula that gives the total mass of a straight one-dimensional object that has a varying density function $\delta(x)\,\frac{\text{g}}{\text{cm}}$ where x is measured in centimeters along an interval $[a, b]$. Use our three-step process for building your definite integral.

2. Suppose now that you are working with a wire in the shape of a semicircle of radius R that lies along the graph of the parametrized curve $x = R\cos(\theta)$, $y = R\sin(\theta)$, $0 \leq \theta \leq \pi$. Suppose also that you know the density of this wire at the point along the semicircle making an angle θ with the positive x-axis is given by $\delta(\theta) = k\sin(\theta)\,\frac{\text{g}}{\text{cm}}$ where k is a constant. Use our three-step process to find a definite integral that gives the total mass of this wire. Note that your answer to the previous question does not apply since the wire is not straight.

3. In order to design a model of the flow of blood through a blood vessel, such as a vein or an artery, it is reasonable to assume the shape of a modeled blood vessel to be a cylindrical tube with radius R and length L. Because of friction at the walls of an artery or vein, it is also reasonable to assume the velocity v (measured in meters per second) of the blood is greatest along the central axis of the tube and decreases as the distance r from the axis increases until v becomes 0 at the wall. The relationship between v and r is given by the law of laminar flow first described by Jean Poisseuille (1799-1869):

$$v = \frac{P}{4nL}\left(R^2 - r^2\right)$$

   where n is the viscosity of the blood and P is the pressure difference between the ends of the tube. If P and L are constant, then v is a function of r with domain $[0, R]$.

   Use our three-step procedure to build a definite integral that computes the flux (volume of blood that crosses a given cross section of the blood vessel per unit time). To do so you should begin by partitioning the interval $[0, R]$ and use this partition to think of the interior of the blood vessel as a collection of nested cylindrical shells. Then estimate the amount of blood in each shell that passes a given cross section of the blood vessel.